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IMPROVED DIFFUSION

by

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ABSTRACT

Plasma effects are shown to decrease both the magnitude of the transverse photon energy density and the rate of diffusion of this energy.

This paper is a partial summary of a talk presented at the Conference on Hot Dense Matter
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I. INTRODUCTION

Although diffusion codes have been in use for a long time, most of them ignore plasma effects which can significantly reduce the rate of energy transport in the transverse photon field. This reduction is important when $\hbar\omega_o/kt > .1$, where ω_o is the plasma radius frequency, k is Boltzmann's constant, T is the absolute temperature and $2\pi\hbar$ is Planck's constant. Since $\omega_o^2 = \frac{4\pi Ne^2}{m}$, (where N is the free electron density, e is the electron charge and m is the electron mass), the reduction is important for high densities and/or low temperatures. Unless these plasma effects are included, even exact frequency dependent opacities cannot produce correct diffusion results.

In order to demonstrate the importance of plasma effects, a simple approximation to the index of refraction is assumed. Modifications in energy density, group velocity, and Rosseland means are noted. A few comments concerning a more accurate index of refraction are made at the end of this note.

II. MODIFICATION OF ENERGY DENSITY

The energy density in a blackbody radiation field in a material with index of refraction, n, is:

$$U(\omega) = \frac{\hbar\omega}{e^{\hbar\omega/kt} - 1} \; \frac{\omega^2 n^2}{\pi^2 c^3} \; \frac{d(n\omega)}{d\omega} \tag{1}$$

where c is the speed of light in a vacuum. An exact expression for n at high densities has yet to be derived. A reasonable expression for n involves an integral over the oscillator strengths of the material of interest. For the purposes of this note, n^2 can be approximated by $1 - \omega_o^2/\omega^2$. In this approximation, (1) becomes

$$U(\omega) = \frac{1}{\pi^2 c^3} \frac{\hbar\omega^3}{e^{\hbar\omega/kt}-1} (1 - \omega_o^2/\omega^2)^{1/2} \tag{2}$$

The only differences between (2) and the blackbody radiation energy density in a vacuum are the factors involving the square root and the absence of frequencies below ω_o in the plasma. Eq. 2 is plotted in Fig. 1 with and without this square root factor. In a material with $n = (1 - \omega_o^2/\omega^2)^{1/2}$, waves with $\omega < \omega_o$ are evanescent and do not propagate; waves with $\omega > \omega_o$ have a decreased energy density. The area between the two curves in Fig. 1 represents the difference in energy density between a vacuum and a plasma. For Fig. 1, it was assumed that $\hbar\omega_o/kt=2$. In Fig. 2, the frequency-integrated difference is plotted as a function of $\hbar\omega_o/kt$. This figure demonstrates that the equilibrium energy density in a plasma should not be assumed to be the same as the equilibrium energy density in a vacuum. In diffusion, the energy transported is proportional to the energy difference between two zones. If this energy difference is calculated for a vacuum instead of a plasma, the result can be incorrect by an order of magnitude.

The improved energy density (2) can be separated into a part associated with the electric field, a part associated with the magnetic field, and a part associated with the electron kinetic energy.[2] This separation is especially interesting to those who like to keep track of electron energy. This electron energy associated with transverse electromagnetic waves should be added to the electron energy which is usually included in an equation of state.

III. MODIFICATION OF THE PHOTON GROUP VELOCITY

The group velocity is defined as $v_g = d\omega/dk$. The dispersion relation for transverse electromagnetic waves in a plasma is

$$\omega^2 = \omega_o^2 + k^2c^2. \quad (3)$$

Hence the electromagnetic waves have a group velocity of

$$c(1 - \omega_o^2/\omega^2)^{1/2}. \quad (4)$$

This requires that the waves travel slower through a plasma than they would through a vacuum.

IV. MODIFICATION OF THE ROSSELAND MEAN

The above ideas can be incorporated into a calculation of modified Rosseland means in such a way that they can be used by diffusion codes to obtain better results. The procedure to do this using $n^2 = 1 - \omega_o^2/\omega^2$ appears to have been proposed first by Aharony and O ner[1]. Their ratio of modified to unmodified Rosseland means is

$$\frac{k_p}{k} = \frac{\displaystyle\int_o^\infty \frac{\omega^4 \exp(\hbar\omega/kt)d\omega}{k_\omega[\exp(\hbar\omega/kt) - 1]^2}}{\displaystyle\int_{\omega_o}^\infty \frac{\omega^4(1-\omega_o^2/\omega^2)\exp(\hbar\omega/kt)d\omega}{k_\omega[\exp(\hbar\omega/kt) - 1]^2}} \quad (5)$$

All of the effects mentioned above are included: modified energy density, modified group velocity and shift of the lower bound of integration. However, all of the above assumes an approximate form of the index of refraction; namely, $n^2 = 1 - \omega_o^2/\omega^2$.

To calculate correctly energy densities, group velocities and Rosseland means, the frequency-dependent index of refraction or dielectric constant must be calculated from first principles. This calculation has not been completely formulated, although various approximations have been proposed.[4] Certainly the index of refraction will depend on the atomic composition of the material and the material temperature as well as the material density. Hence, a general, material independent formulation is not possible. Detailed calculations are required for each material. It is hoped that consistent procedures can be developed soon so that plasma effects on transport properties can be accurately modeled.

V. REFERENCES

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2. G. Bekefi, Radiation Processes in Plasmas, John Wiley & Sons 1966, p. 111.
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FIGURE 1

Equilibrium Radiation Energy Density

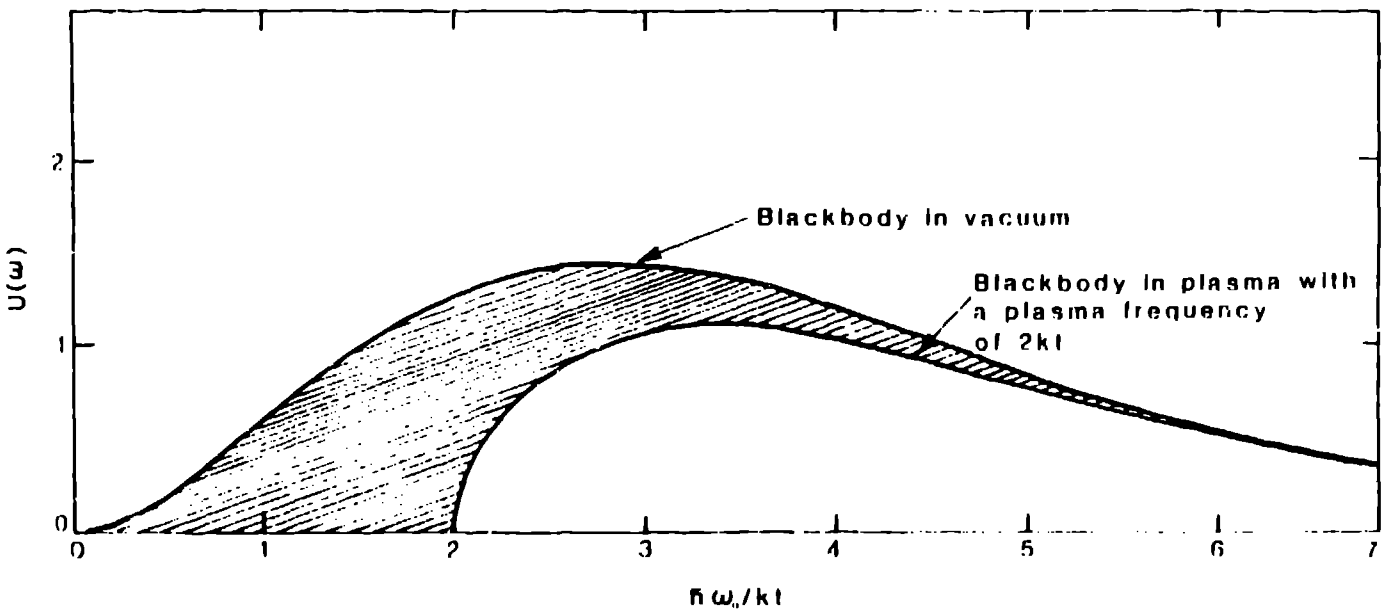

FIGURE 2

Fraction of Vacuum Blackbody Radiation Absent in a Plasma